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HEAT TRANSFER AT A MELTING FLAT SURFACE UNDER CONDITIONS OF FORCED CONVECTION AND LAMINAR BOUNDARY LAYER

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Аннотация—Рассматривается теплообмен на плавящейся поверхности. Задача решается методом Кармана-Польгаузена с использованием физических особенностей процесса плавления. С помощью полученных зависимостей на ЭВЦМ рассчитан теплообмен на плоской стенке в системах лед-вода и т вердый-жидкий этиленгликоль. Результаты решения показывают, что влияние плавления на теплообмен определяется критерием Кутателадзе и Прандтля. Показано, что параметр плавления $(v_w/W)\sqrt{Pe_x}$ не зависит от гидродинамических условий, а является функцией только k_f и Pr_f. Рекомендовона формула для инженерный расчетов теплообмена на плоской плавящейся стенке.

NOMENCLATURE

Nu, Nusselt number for heat transfer at melting surface;
Nu_0, Nusselt number for heat transfer at non-melting surface;
$\lambda_0 = (v_w . \delta/6a_f)$, parameter describing melting effect on heat transfer;
v_w, flow rate in the boundary layer of the new phase resulting from melting;
δ, thickness of thermal boundary layer;
k, thickness of hydrodynamic boundary layer;
a_f, thermal diffusivity of liquid;
v_f, kinematic viscosity of liquid;
k_f $= (r/c_f . \theta_f)$, phase transition criterion (Kutateladze number);
r, latent heat of melting;
γ_f, specific weight of liquid;
Pe_x $= (Wx/a_f)$, Péclét number defined by the characteristic dimension x;
W, main flow velocity;
θ_f, temperature of the main flow referred to the melting temperature of the wall material;
$(v_w/W)\sqrt{Pe_x}$, melting parameter;
λ_f, thermal conductivity of liquid;
c_f, specific heat;
Nu_x, $= (\alpha x/\lambda_f)$;
Nu, mean Nusselt number over the melting surface of a sphere;
Nu_0, mean Nusselt number over the non-melting surface of a sphere.

HEAT transfer at a flat melting surface under conditions of forced convection and laminar boundary layer was investigated earlier by Tkachev [1] who gave the following solution of the problem considered

$$\frac{Nu}{Nu_0} = f(\lambda_0, Pr_f).$$

This relation is not convenient for practical calculations of transfer process as it is difficult to calculate the parameter λ_0 exactly. The present solution which considers physical peculiarities of the melting process allows results to be obtained which may be used in engineering practice.

Plot of the temperatures and velocity distributions in a boundary layer under the given problem conditions is shown in Fig. 1. The liquid with velocity W and temperature θ_f, referred to the melting temperature, flows to the melting plate set along the coordinate x.

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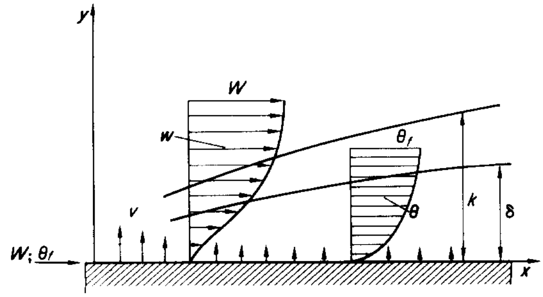

FIG. 1. Plot of temperature and velocity distributions in the boundary layer.

Assuming thermophysical properties of the liquid to be independent of the temperature, we may write the boundary layer equations for the steady-state conditions as follows

$$w\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \nu_f\frac{\partial^2 w}{\partial y^2} \tag{1}$$

$$\frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$w\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = a_f\frac{\partial^2 \theta}{\partial y^2}. \tag{3}$$

The boundary conditions are

$$\begin{aligned} &\text{at} \quad y = 0; \quad v = v_w; \quad w = 0 \\ &v_w\left(\frac{\partial w}{\partial y}\right)_{y=0} = \nu_f\left(\frac{\partial^2 w}{\partial y^2}\right)_{y=0} \\ &\theta = 0; \quad v_w\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = a_f\left(\frac{\partial^2 \theta}{\partial y^2}\right)_{y=0} \end{aligned} \tag{4}$$

$$\begin{aligned} &\text{at } y = k: \quad w = W \\ &\frac{\partial^2 w}{\partial y^2} = \frac{\partial w}{\partial y} = 0 \\ &\theta = \theta_f \frac{\partial^2 \theta}{\partial y^2} = \frac{\partial \theta}{\partial y} = 0. \end{aligned} \tag{4'}$$

The heat-transfer equation at the phase interface describes the physical peculiarities of heat transfer in melting

$$v_w \cdot \gamma_f \cdot r = \lambda_f\left(\frac{\partial \theta}{\partial y}\right)_{y=0}. \tag{5}$$

Integrating equations (1)–(3) across the boundary layer and using equation (5) gives the following relations

$$W\frac{\mathrm{d}}{\mathrm{d}x}\left[\delta\int_0^1\left(1 - \frac{\theta}{\theta_f}\right)\frac{w}{W}\,\mathrm{d}\eta\right] = \frac{a_f}{\theta_f\delta}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}\left(1 + \frac{1}{k_f}\right) \tag{6}$$

$$W\frac{\mathrm{d}}{\mathrm{d}x}\left[k\int_0^1\left(1 - \frac{w}{W}\right)\frac{w}{W}\,\mathrm{d}\eta_l\right] - v_w = \frac{\nu_f}{Wk}\left(\frac{\partial w}{\partial \eta_l}\right)_{\eta_l=0}. \tag{7}$$

Or assuming that k/δ is independent of x, we obtain

$$\delta^{**}\frac{\mathrm{d}\delta}{\mathrm{d}x} = \frac{a_f}{\theta_f w\delta}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}\left(1 + \frac{1}{k_f}\right) \tag{8}$$

$$k^{**}\frac{\mathrm{d}k}{\mathrm{d}x} = \frac{\nu_f}{W^2 k}\left(\frac{\partial w}{\partial \eta_l}\right)_{\eta_l=0} + \left(\frac{v_w}{W}\right) \tag{9}$$

where

$$\delta^{**} = \int_0^1\left(1 - \frac{\theta}{\theta_f}\right)\frac{w}{W}\,\mathrm{d}\eta$$

$$k^{**} = \int_0^1\left(1 - \frac{w}{W}\right)\frac{w}{W}\,\mathrm{d}\eta_l$$

$$\eta = \frac{y}{\delta}; \quad \eta_l = \frac{y}{k}.$$

The velocity and temperature distribution in a boundary layer is presented as follows

$$w = W(a'\eta_l + b'\eta_l^2 + c'\eta_l^3 + d'\eta_l^4) \tag{10}$$

$$\theta = \theta_f(a\eta + b\eta^2 + c\eta^3 + d\eta^4). \tag{11}$$

Agreement of equations (10) and (11) with boundary conditions (4) gives the following distributions of the velocity and temperature in the boundary layer

$$w = W\left(\frac{2}{1+\lambda_0'}\eta_l + \frac{6\lambda_0'}{1+\lambda_0'}\eta_l^2 - \frac{2+8\lambda_0'}{1+\lambda_0'}\eta_l^3 + \frac{1+3\lambda_0'}{1+\lambda_0'}\eta_l^4\right) \quad (10a)$$

$$\theta = \theta_f\left(\frac{2}{1+\lambda_0}\eta + \frac{6\lambda_0}{1+\lambda_0}\eta^2 - \frac{2+8\lambda_0}{1+\lambda_0}\eta^3 + \frac{1+3\lambda_0}{1+\lambda_0}\eta_l^4\right) \quad (11a)$$

where

$$\lambda_0' = \frac{v_w k}{6 v_f}; \qquad \lambda_0 = \frac{v_w \delta}{6 a_f}.$$

Thus

$$a' = \frac{2}{1+\lambda_0'}; \qquad b' = \frac{6\lambda_0'}{1+\lambda_0'};$$

$$c' = -\frac{2+8\lambda_0'}{1+\lambda_0'}; \qquad d' = \frac{1+3\lambda_0'}{1+\lambda_0'};$$

$$a = \frac{2}{1+\lambda_0}; \qquad b = \frac{6\lambda_0}{1+\lambda_0};$$

$$c = -\frac{2+8\lambda_0}{1+\lambda_0}; \qquad d = \frac{1+3\lambda_0}{1+\lambda_0}.$$

Integration of equation (8) with distributions (10a) and (11) results in

$$\frac{\delta}{x} = \sqrt{\left(\frac{2a}{\delta^{**}} \quad \frac{1+1/k_f}{Pe_x}\right)}. \quad (12)$$

Bearing in mind that

$$Nu_x = \frac{x}{\theta_f}\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = a\frac{x}{\delta}$$

we have

$$Nu_x = \sqrt{\left(\frac{a}{2}\cdot\frac{1}{(1+1/k_f)}\right)}(\sqrt{\delta^{**}})\sqrt{Pe_x}. \quad (13)$$

In the case of heat transfer without melting $a = 2$, $k_f \to \infty$; $\delta^{**} = \delta_0^{**}$ and the Nusselt number will be equal to

$$Nu_{x_0} = (\sqrt{\delta_0^{**}})\cdot\sqrt{Pe_x} \quad (14)$$

hence

$$\frac{Nu_x}{Nu_{x_0}} = \sqrt{\left(\frac{a}{2}\frac{1}{1+1/k_f}\right)}\sqrt{\left(\frac{\delta^{**}}{\delta_0^{**}}\right)}. \quad (15)$$

The values a and λ_0 are defined from equation (5) using distribution (11)

$$a = 3\,[\sqrt{(k_f^2 + \tfrac{4}{3}k_f)} - k_f] \quad (16)$$

$$\lambda_0 = \frac{a}{6k_f}. \quad (17)$$

These values are plotted in Fig. 2 vs. the Kutateladze number. Values δ^{**} and δ_0^{**} depending on the ratio k/δ are included into equation (15).

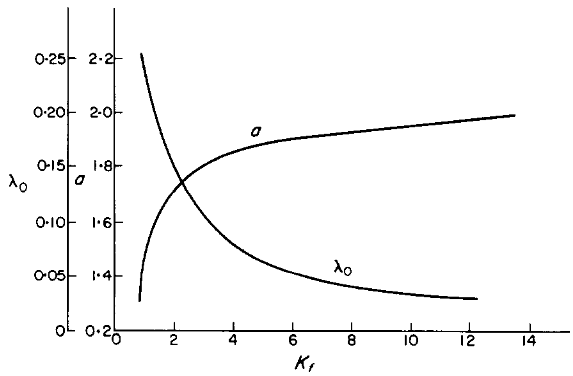

FIG. 2. Plot of a and λ_0 vs. the Kutateladze number.

The ratio k/δ is determined from (8) and (9)

$$\frac{k}{\delta} = \sqrt{\left(\frac{\delta^{**}}{k^{**}}\right)}\cdot\frac{\sqrt{\left(\frac{A}{a}\right)}}{\sqrt{\left(1+\frac{1}{k_f}\right)}}\cdot\sqrt{Pr_f} \quad (18)$$

where

$$A = a' + 6\lambda_0' = A(Pr_f, k_f, k/\delta)$$
$$\delta^{**} = \delta^{**}(k_f, Pr_f, k/\delta)$$
$$k^{**} = k^{**}(k_f, Pr_f, k/\delta).$$

For the case of no melting

$$\frac{k_0}{\delta_0} = \sqrt{\left(\frac{\delta_0^{**}}{k_0^{**}}\right)}\cdot\sqrt{Pr_f}. \quad (18a)$$

The values of the dimensionless Nusselt number Nu_x/Nu_{x_0} as a function of the Kutateladz number for water and ethylene glycol and parameters defining these values were calculated on an electronic digital computer using equations (15) and (18). These calculations are shown in Table 1.

Table 1.

k_f	Pr_f	k/δ	δ^{**}	k^{**}	δ_0^{**}	δ_0/k_0	$\frac{Nu_x}{Nu_{x_0}}$	Material
1	2·23	1·2574	0·1013	0·1188	0·0930	0·7526	0·6564	water
2	4·30	1·6612	0·0790	0·1180	0·0760	0·5731	0·7490	
5	7·91	2·0032	0·0654	0·1176	0·0627	0·4866	0·9045	
10	10·27	2·2162	0·0591	0·1175	0·0577	0·4452	0·9496	
1	49·5	3·4060	0·0421	0·1177	0·0346	0·2620	0·6942	ethylene glycol
2	147·7	5·1440	0·0273	0·1175	0·0241	0·1816	0·8118	
5	458·0	7·7938	0·0175	0·1175	0·0165	0·1245	0·9119	
10	694·0	9·0848	0·0149	0·1175	0·0144	0·1083	0·9530	

Transforming equation (15) yields the following interesting relation

$$\frac{v_w}{W}(\sqrt{Pe_x}) = \frac{\sqrt{(\delta_0^{**})}}{k_f} \cdot \frac{Nu_x}{Nu_{x_0}}. \qquad (19)$$

This relation shows that the group $v_w/W\sqrt{Pe_w}$, the physical significance of which is a melting parameter, is at the same time a quantity independent of the hydrodynamic conditions of the problem considered. The dimensionless Nusselt number for water and ethylene glycol is plotted vs. the melting parameter in Fig. 3.

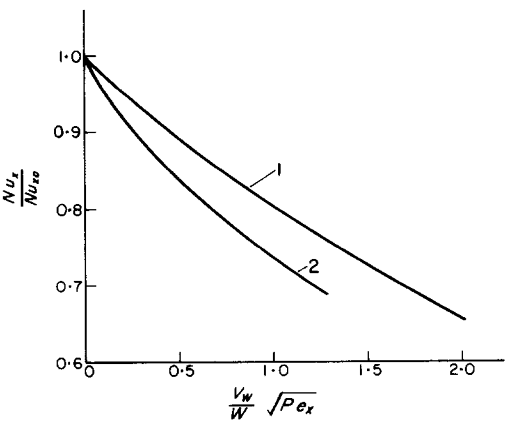

FIG. 3. Plot of the dimensionless Nusselt number vs. the parameter $v_w/W\sqrt{Pe_x}$.
1 : water ; 2 : ethylene glycol.

The calculation results presented in Table 1 are well approximated by the following relations

$$\delta_0^{**} \cong 0{\cdot}124\, Pr_f^{-\frac{1}{3}}$$

$$\delta^{**} \cong 0{\cdot}124 \left(\frac{1}{1 + \frac{a}{6k_f} \cdot \frac{k/\delta}{Pr_f}} \right) Pr_f^{-\frac{1}{3}}$$

$$\frac{k_0}{\delta_0} \cong 1{\cdot}05\, Pr_f^{\frac{1}{3}}$$

$$\frac{k}{\delta} \cong 1{\cdot}05 \left[\frac{2}{a} \cdot \frac{1}{(1 + 1/k_f)} \right]^{\frac{1}{3}} Pr_f^{\frac{1}{3}}.$$

Furthermore, the calculation analysis has shown that the dimensionless Nusselt number may be represented with an accuracy of 8 per cent by the group

$$\sqrt{\left(\frac{a}{2} \cdot \frac{1}{(1 + 1/k_f)}\right)},$$

i.e. the dimensionless Nusselt number is determined mainly by the Kutateladze number.

Thus, the following relation may be recommended for the practical calculations of heat transfer on a flat melting wall under conditions of forced convection and laminar boundary layer

$$\frac{Nu}{Nu_{x_0}} \cong \sqrt{\left(\frac{a}{2} \cdot \frac{1}{(1 + 1/k_f)}\right)}$$
$$k_f = 1 \div \infty.$$

The same method was used to solve the problem of heat transfer of a melting sphere in a forced liquid flow (laminar boundary layer). The analysis of the solution has shown that in this case the following relation is valid to within 10 per cent

$$\frac{\overline{Nu}}{\overline{Nu}_{x_c}} = \sqrt{\left(\frac{a}{2} \cdot \frac{1}{(1 + 1/k_f)}\right)}.$$

To verify the validity of the latter relation, experimental investigations of heat transfer at the surface of an ice sphere melting in a water flow $Re_s = (50 \div 1050)$ were carried out. Re_s is the Reynolds number determined by the initial diameter of the melting sphere.

Comparison of the experimental results (individual points are the arithmetic means of 20 identical runs) with the theoretical curve obtained for heat transfer at a melting sphere is shown in Fig. 4.

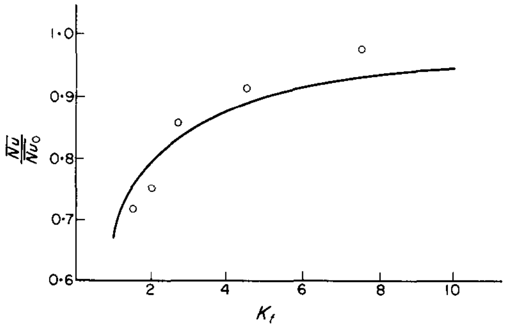

FIG. 4. Comparison of experimental results for heat transfer at a melting sphere of ice with theoretical solution.

The Nusselt number $\overline{Nu}_0$ was determined with a hollow metal sphere through which cold water was pumped ($\sim 0°C$).

REFERENCE

1. A. G. TKACHEV, Convective heat transfer in the melting and solidification processes of homogeneous medium, Doctor Degree Thesis, Leningrad (1955).

Abstract—Heat transfer at a melting surface is considered. The problem is solved by the Kàrmàn–Pohlhausen method including physical peculiarities of the melting process. Heat transfer at a flat wall for the systems ice–water and solid–liquid ethylene glycol was calculated on the electronic digital computer with the help of the relations obtained. The results of the solution show that the melting effects on heat transfer are determined by the Kutateladze and Prandtl numbers. It is shown that the melting parameter $v_w/W\sqrt{}Pe_x$ does not depend on the hydrodynamic conditions and is a function of k_f and Pr_f only. An equation for practical calculation of heat transfer at a flat melting surface is presented.

TRANSFERT DE CHALEUR SUR UNE SURFACE PLATE FONDANTE DANS DES CONDITIONS DE CONVECTION FORCÉE ET DE COUCHE LIMITROPHE LAMINAIRE

Résumé—On considère le transport de chaleur sur une surface en fusion. Le problème est résolu par la méthode de Kármán–Pohlhausen en tenant compte des particularités physiques du processus de fusion. Le transport de chaleur sur une paroi plane pour les systèmes glace–eau et l'éthylène-glycol solide et liquide est calculé sur un calculateur numérique électronique à l'aide des relations obtenues. Les résultats de la solution montrent que les effets de la fusion sur le transport de chaleur sont déterminés par les nombres de Kutateladze et de Prandtl. On montre que le paramètre de fusion $v_w\sqrt{(Pe_x)}/W$ ne dépend pas des conditions hydrodynamiques et est une fonction seulement de K_f et de Pr_f. On recommande pour des calculs pratiques la formule du transport de chaleur sur une surface plane en fusion.

WÄRMEÜBERGANG AN EINER SCHMELSEUDEN OBERFLÄCHE BEI ZURANGSKANVEKTION UND LAMINAUER GRENSCHICHT

Zusammenfassung—Es wird der Wärmeübergang an einer schmelzenden Oberfläche behandelt. Das Problem wird nach der Kármán–Pohlhausen-Methode unter Berücksichtigung der physikalischen Eigenheiten des Schmelzprozesses gelöst. Der Wärmeübergang an einer ebenen Wand wird für die Systeme Eis–Wasser und für fest–flüssiges Äthylenglykol mit Hilfe der erhaltenen Beziehungen auf einem elektronischen Digitalrechner berechnet.

Die Ergebnisse der Lösung zeigen, dass die Einflüsse des Schmelzens auf den Wärmeübergang von der Kutateladse- und Prandtl-Zahl bestimmt werden.

Der Schmelzparameter $(v_w/W)\sqrt{Pe_x}$ hängt nicht von hydrodynamischen Bedingungen ab und ist nur eine Funktion von K_f und Pr_f. Die Gleichung für die praktische Berechnung des Wärmeübergangs an einer ebenen schmelzenden Oberfläche wird empfohlen..